

SECTION 3

Solving Quadratic Equations and Inequalities

Solving Quadratics by Factoring

What You Need to Know

The factoring we practiced in Section 2 is useful for solving quadratic equations. We use the Zero Product Property, which states that when a product is equal to 0, at least one of the factors must equal 0. We note here that to solve quadratic inequalities is a bit more complex; we suggest the use of sign charts to analyze the situation in a visual way.

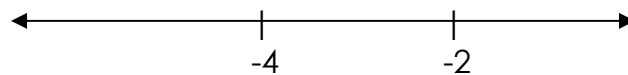
Example 1: Solve $x^2 - 11x + 18 = 0$.

We factor the quadratic expression.
We set each factor equal to 0.
We solve to obtain...

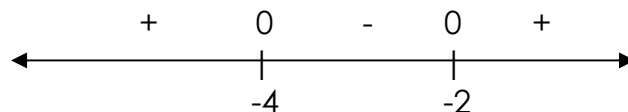
$$\begin{aligned}(x - 2)(x - 9) &= 0 \\ x - 2 = 0 \text{ or } x - 9 &= 0 \\ x = 2 \text{ or } x &= 9\end{aligned}$$

Example 2: Solve $x^2 + 6x + 8 > 0$.

As before, we factor the quadratic expression to obtain $(x + 4)(x + 2) > 0$. However, we cannot apply the Zero Product Property, since this is an inequality and not an equation. We use a number line, and we mark the values of x for which the expression would be equal to 0.



We now consider the sign of the expression $(x + 4)(x + 2)$ in each subinterval of the number line. Simply substituting a value of x from each subinterval tells us that the value of the expression has the following sign chart:



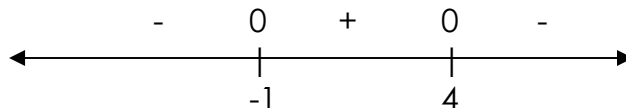
We want the quadratic expression to be greater than 0, so we choose the positive portions of the number line. Our solution is $\{x \mid x < -4 \text{ or } x > -2\}$ or $(-\infty, -4) \cup (-2, \infty)$.

Example 3: Solve $-x^2+3x \geq -4$.

First, we manipulate the inequality so that one side is 0: $-x^2 + 3x + 4 \geq 0$

Then, we factor the left side of the inequality. $-(x - 4)(x + 1) \geq 0$

We set up our number line, as before, marking the two values of x where the value of the quadratic expression is equal to 0, and fill in the number line by substituting values of x from each subinterval on the number line.



Since the quadratic must be non-negative, our solution is $[-1, 4]$ or $\{x \mid -1 \leq x \leq 4\}$.

Solving Quadratic Equations by Completing the Square

**What You
Need to
Know**

Some quadratic equations cannot be factored, so solving their associated equations is more difficult. We can solve these equations through a process known as completing the square. Essentially, we put the equation of a parabola in “vertex form” and then use a square root to solve.

Example 4: Solve $x^2 - 6x - 9 = 0$.

First, we move the constant term to the right side.

$$x^2 - 6x = 9$$

Then, we *complete the square*. This is done by squaring half the coefficient of the “ x ” term and adding this number to both sides. In this case, $(-6/2)^2 = 9$, so...

$$x^2 - 6x + 9 = 18$$

Why did we do that? Look how the left side factors:

$$(x - 3)^2 = 18$$

We now take the square root of both sides:

$$x - 3 = \pm\sqrt{18}$$

Adding 3 to both sides yields our solution:

$$x = 3 \pm \sqrt{18}$$

Example 5: Solve $3x^2 + 6x - 7 = 0$.

As before, we move the constant term.

$$3x^2 + 6x = 7$$

We take the extra step of dividing by the coefficient of x^2 to make the problem analogous to Example 4:

$$x^2 + 2x = \frac{7}{3}$$

We complete the square. Here, we have $(2/2)^2 = 1$...

$$x^2 + 2x + 1 = \frac{10}{3}$$

Rewrite the left side:

$$(x + 1)^2 = \sqrt{\frac{10}{3}}$$

We take a square root and solve:

$$x = -1 \pm \sqrt{\frac{10}{3}}$$