

Discrete Probability Distributions

A probability distribution shows the probabilities for all the outcomes of a particular event. Discrete probability distributions relate to events which can only have certain outcomes – usually in the form of integers.

Note that the capital letter X is used to describe the random variable, whereas lower case x is used to represent the actual values.

Uniform distributions: If all the outcomes are equally likely, the distribution is called *uniform*. For example, here is the probability distribution for the random variable X where X represents the outcomes when throwing a die.

x	1	2	3	4	5	6
$P(X = x)$	1/6	1/6	1/6	1/6	1/6	1/6

Distributions defined by a function: The following is an example of a probability distribution:

$$P(X = x) = \begin{cases} kx, & x = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

This means that x can only take values 1 to 5, and has probability kx for these values. The best thing to do is put all the information into a table:

x	1	2	3	4	5
$P(X = x)$	k	$2k$	$3k$	$4k$	$5k$

In all probability distributions, the probabilities add to give 1, so $15k = 1$, giving $k = \frac{1}{15}$. We can fill the probabilities into the table:

x	1	2	3	4	5
$P(X = x)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

Expected values of distribution parameters: We can now calculate what we expect to happen when the trials take place. The more trials that are carried out, the closer the observed statistics will get to the expected ones.

- **Expected mean (expectation)** = $\sum xp$. Simply multiply each pair of values along the table. In the example, this gives $\frac{55}{15} = 3.67$
- **Expected variance** = $\sum x^2p - \bar{x}^2$. Remember this as the "expectation of the squares – the square of the expectation." Work through the example; you should get 15.
- **Expected standard deviation** is the square root of the expected variance; in this case, $\sqrt{15} = 3.87$.
- **Expected mode** is the value with the highest probability: 5.
- **Expected median** is the value which you have an equal probability of getting above or below. We can see from the table that the 50:50 split comes when $x = 4$. ($\frac{7\frac{1}{2}}{15}$ both sides!)

YOU SOLVE

The probability distribution for a random variable x is given by:

$$P(X = x) = kx(x - 1), \text{ for } x = 2, 3, 4, 5, 6$$

- Find the value of k .
- Find the expected mean and variance of the distribution.

Begin by drawing up a probability table.

$$k = 1/70, \quad E(X) = 5, \quad \text{var}(X) = 1.2$$