Notes and jottings column

Points to note:

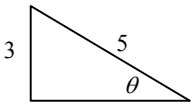
- The range of both functions is $-1 \le f(x) \le 1$
- $\sin x > 0$ for angles between 0° and 180°
- $\cos x > 0$ for angles between 0° and 90°, also between 270° and 360°
- Both functions have a *period* (ie repeat themselves) every 360°.

Simple trigonometric equations: $\sin\theta = 0.4$, $0° \le \theta \le 360°$. What is the value of θ? We want to know what angle has a sin which is 0.4. Using the inverse of the sin function (written as $\sin^{-1}$ or arcsin) on your calculator, we find θ is 23.6°. Using the symmetry of the sin graph above, another solution is 180 – 23.6 = 156.4°. (If the domain is in radians, you can either work in degrees and convert at the end, or set your calculator to radians: this gives $\theta = 0.412$, and the second solution is $\pi - 0.411 = 2.73$).

Another example: Solve $\cos(\theta - 30) = 0.2$, $0° \le \theta \le 360°$

$\cos^{-1}(0.2) = 78.5°$ or $281.5°$
So $\theta - 30 = 78.5$ or 281.5
$\theta = 108.5°$ or $311.5°$

Finding sin from cos (and cos from sin): A simple trick is to draw a right-angled triangle. eg If $\sin\theta = \frac{3}{5}$, what is $\cos\theta$? Having put 3 as the "opposite" and 5 as the hypoteneuse, the remaining side must be 4. So $\cos\theta = \frac{4}{5}$. If θ was obtuse, $\cos\theta$ would be $-\frac{4}{5}$.



Trigonometric identities: The identities on the right are on your formulae sheets but should be memorised.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

If A is an obtuse angle in a triangle and $\sin A = \frac{5}{13}$, calculate the exact value of sin2A.

sin2A = 2sinAcosA, so we need cosA. Using the little trick above, draw a 5, 12, 13 triangle. Hence, cosA = - 12/13.

So, sin2A = 2 × 5/13 × (-12/13) = **- 120/169** (Note the answer has to be exact).

Solve the equation $3\sin^2 x = \cos^2 x$, for $0° \le x \le 180°$.

First, get everything in terms of $\sin^2 x$, then make $\sin^2 x$ the subject. When you square root, remember the ±. This will effectively give you two equations to solve. But note the domain.

YOU SOLVE

$x = 30°$ or $150°$

Find the exact solutions to the equation $\sin 2x = \sin x$, for $0 \le x \le 2\pi$

$\sin 2x = \sin x$
$2\sin x\cos x = \sin x$
$2\sin x\cos x - \sin x = 0$ *(as with quadratics, it is important to get 0 on the right hand side)*
$\sin x(2\cos x - 1) = 0$
So $\sin x = 0$ or $2\cos x - 1 = 0 \Rightarrow \cos x = ½$

If $\sin x = 0$, $x = 0°, 180°, 360°$. If $\cos x = ½$, $x = 60°$ or $300°$

So, in radians, **$x = 0, \pi/3, \pi, 5\pi/3, 2\pi$**