Standard Error of the Mean

Notes and jottings column

Random sample: When a population is too big to record data on each of its members, we must take *samples* from the population. It is necessary to ensure that a given sample is truly random (that is, each member of the population is equally likely to be chosen for the sample). One way of doing this is by using tables of random numbers, or using the random number generator on a calculator.

Having obtained our random sample, we can use the mean of the sample as an estimate of the mean of the population as a whole.

Distribution of the sample mean: If you take lots of samples, each one is likely to have a different mean. However, it turns out that *the sample means are distributed normally.* The mean and standard deviation of this distribution are related to the mean and standard deviation of the original population as follows:

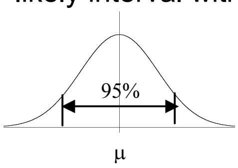
Population: $X \sim N(\mu, \sigma^2)$

Sample means: $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

where n is the sample size.

The standard deviation of the sample means, $\frac{\sigma}{\sqrt{n}}$ is called "the standard error of the mean."

Confidence intervals: If we take one sample, its mean is therefore a *point estimate* for the population mean. Since this value is part of a known normal distribution, we can calculate the likely interval within which the population mean actually lies.



Suppose we want to find the range such that there is only a 5% chance of the mean lying outside this range (ie 2.5% at each end), the tables tell us that Z must lie within 1.96 standard deviations of μ. The 95% is known as a *confidence interval*. Other common confidence intervals are 90% and 99%.

- For 90%, Z must lie within $\mu \pm 1.645$ standard deviations
- For 99%, Z must lie within $\mu \pm 2.58$ standard deviations

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The IQ's of students in a school are known to be normally distributed with a standard deviation of 8.5. A sample of 50 students is found to have a mean IQ of 122. Find a 95% confidence interval for the mean IQ of all students in the school.

The point estimate for μ is 122. The standard error of the mean is $\frac{\sigma}{\sqrt{n}} = \frac{8.5}{\sqrt{50}} = 1.202$. So the 95% confidence interval is given by:

$\mu = 122 \pm 1.96 \times 1.202$ or **<u>$119.6 \le \mu \le 124.4$</u>**

The lives of light bulbs are normally distributed with variance 1500 hours. A sample is to be taken to estimate the mean life. What is the smallest sample size which leads to a 99% confidence level that the error in estimating the mean is less than 10 hours.

Remember that variance is the square of standard deviation. Always check which one has been given in a question

The *error* is $2.58 \times \frac{\sigma}{\sqrt{n}}$, so $2.58 \times \frac{\sqrt{1500}}{\sqrt{n}} > 10$ and this solves to give $n > 99.8$ so ***<u>n = 100</u>***