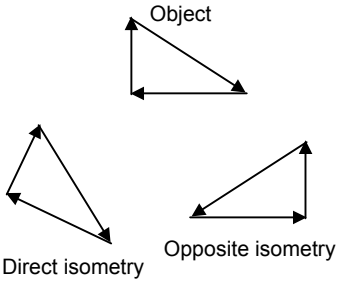
Transformations

Notes and jottings column

There are seven transformations you need to know about: reflection, rotation, translation, enlargement, stretch, shear and glide reflection. Although in practice you are asked to transform points, lines and shapes, it helps to understand that transformations work on the whole *plane*. The various shapes are drawn on the plane and are transformed at the same time. Note that reflection and glide reflection are slightly different from the rest: all the others require the shape to be moved, distorted and a so on *on* the plane, but for these two the objects are picked up and turned over.

Definitions: You need to know the following:

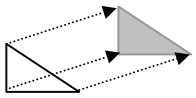
- *Invariant point* – a point which remains unchanged during a transformation.
- *Invariant line* – a line which remains unchanged during a transformation. Note, however, that the points on the line may have moved, but only to other points on the line. If the points on the line are invariant then we have *an invariant line of invariant points.*
- *Isometry* – a transformation which leaves shape and size unchanged. A *direct* isometry leaves the "sense" of the shape unchanged whereas an *opposite* isometry reverses the sense.



The transformations:

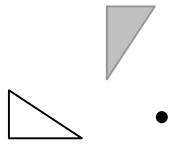
TRANSLATION

- Defined by a vector (how far along, how far up)
- Points "slide" along the same vector
- Direct isometry
- No invariant points



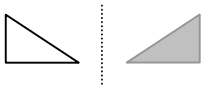
ROTATION

- Defined by a centre and an angle (including direction)
- Points rotate through the same angle about the centre
- Direct isometry
- Centre is the only invariant point



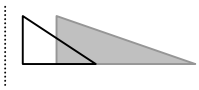
REFLECTION

- Defined by a line (the mirror line)
- The mirror becomes the perpendicular bisector of every point and its image point.
- Opposite isometry
- Mirror line is a line of invariant points



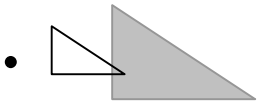
STRETCH

- Defined by an invariant line and a scale factor
- Points move perpendicular to the invariant line and multiply their distance away by the scale factor
- Neither shape nor size preserved, but area multiplied by the scale factor.
- Invariant line is a line of invariant points.



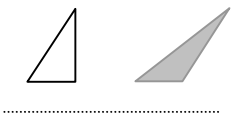
ENLARGEMENT

- Defined by a centre and a scale factor
- Points multiply their distance from the centre by the scale factor
- Shape is preserved, but not size. Area multiplies by (scale factor)^2
- Centre is the only invariant point



SHEAR

- Defined by an invariant line and a scale factor.
- Points move parallel to the invariant line. Distance moved is distance from line $\times$ scale factor.
- Shape not preserved but area remains constant.



GLIDE REFLECTION

- Defined by a mirror line and a distance.
- Points reflect in the line and move parallel to the line by the given distance. (Think of it as a single "corkscrew" motion).
- Opposite isometry.
- Points on invariant line are *not* themselves invariant.