

Basics of Functions

A function is an algebraic rule which shows how one set of numbers is related to, or obtained from another set. Functions often model real-life situations, so it is necessary to understand the notation used and the different types of function which may be used.

Defining functions: A function is defined using the notation $f: x \rightarrow \dots$. For example, $f: x \rightarrow x^2 - 1$. An alternative notation is $f(x) = x^2 - 1$ so that, for example, $f(3) = 3^2 - 1 = 8$. The x value put in to the function is called the *object* and the value of the function which results is called the *image*. Any letters can be used besides f and x .

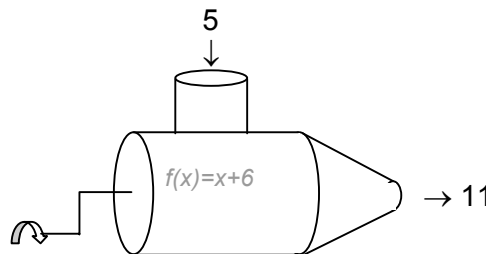
Read the definition as: "The function f takes any number x and turns it into $x^2 - 1$ "

Domain: The set of values to be put into a function is called the *domain* of the function. In many functions, *any* value can be input, in which case the domain is $x \in \mathbb{R}$. However, the domain may be restricted in a particular question. For example, suppose the x numbers in a function are limited to 3, 4 and 5, we can show the domain like this:

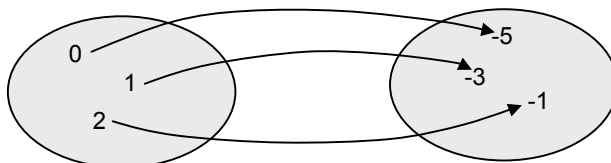
$$f: x \rightarrow 2x - 3, 3 \leq x \leq 5, x \in \mathbb{I}$$

Range: The set of values produced by a function is called the *range*. For example, the function defined by $f: x \rightarrow x^2$ has a range $f(x) \geq 0$ since it is impossible for squares to be negative. Generally, the easiest way to find the range of a function is to look at its graph: the range is the complete set of possible y values.

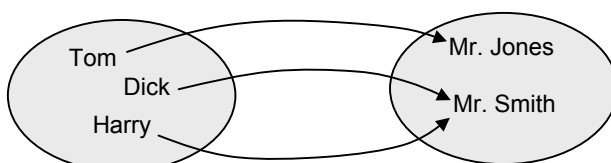
Imagine a "function machine." When the handle is turned, the 5 drops in the top, and the function machine turns it into a 11!



Mapping Diagram: A mapping diagram is a simple way to illustrate how members of the domain are "mapped" onto members of the range. For example, the mapping diagram below shows what happens to the numbers 0, 1 and 2 under the function $f(x) = 2x - 5$.



Functions do not have to be algebraic. For example, the diagram below could represent the function mapping sons onto fathers.



For a relationship to be a function, each member of the domain can only map on to *one* member of the range; but it is OK for different members of the domain to map onto the *same* member of the range. (So fathers \rightarrow sons would not be a function since Mr. Smith has 2 sons).