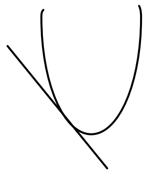
Applications of Differentiation



Equations of tangents: A tangent to a graph has the same gradient as at the point on the graph where the tangent touches. Knowing this, and the point itself, we can find the equation of the tangent. Remember that when you differentiate a function you get the *gradient function*.

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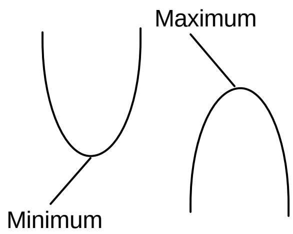
eg: Find the equation of the tangent to $y = 2x^2 - 4x + 3$ at the point where x = 2.

- $\frac{dy}{dx} = 4x - 4$, *so when x = 2, gradient = 4*
- *When x = 2, y = 3*
- *Equation is given by $y - y_1 = m(x - x_1)$, so $y - 3 = 4(x - 2)$*
  Equation of the tangent is $y = 4x - 5$



Maximum and minimum points: The point where a graph "turns round" can be very significant. For example, if the graph shows values of profit against selling price for a particular product, the maximum shows the selling price which leads to maximum profit.

- To find a maximum or minimum, differentiate the function then find where the gradient is 0.
- To tell which sort of point you have, use the second derivative or a sign diagram.
- Note that you do not need the graph in front of you to find the turning points. However, make sure you can use your calculator to find maximum and minimum values (for example, *find the x-coordinates of all maximums and minimums on the graph of f(x) = sin(1 + sinx), 0 £ x £ 6*)

Find the turning point on the graph of $y = \ln(2 + x^2)$, giving coordinates as exact values, and determine whether it is a maximum or minimum.

- $\frac{dy}{dx} = \frac{2x}{2 + x^2}$ (using the chain rule)
- For a turning point, $\frac{dy}{dx} = 0 \Rightarrow 2x = 0 \Rightarrow x = 0$
- So the turning point is at **(0, ln2)**

x	-1	0	1
dy/dx	-2/3	0	2/3

The sign diagram shows the values of $\frac{dy}{dx}$ either side of the turning point. Drawing the gradients is not necessary, but it helps. I chose to use a sign diagram because the second derivative looked rather awkward.

- So (0, ln2) is a **minimum** *(Check by drawing the graph)*

Velocity and acceleration: Since velocity is rate of change of distance, differentiating a distance-time function will give velocity. Similarly, differentiating a velocity-time function will give acceleration (which is the rate at which velocity changes).

YOU SOLVE

A ball is thrown in the air and its height in metres *t* seconds afterwards is given by the formula $h = 20t - 5t^2$. Find when the ball reaches its maximum height, and what this height is.

2s, 20m