

Binomial Distribution

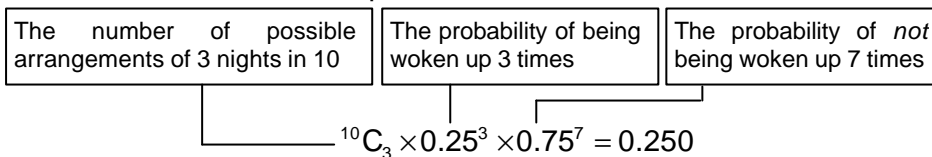
The Binomial probability distribution is a special case of a discrete distribution. You can use it when:

- There are a fixed number of "trials"
- Each trial has only two possible outcomes, "success" and "failure."
- The results of each trial are independent of each other.
- The probability of success remains the same.

For example, my young child wakes me up 1 night in 4. I want to find the probability that I will be woken up 3 nights out of 10.

- The number of trials, n , is 10.
- The probability of "success" (ie being woken up!) is 0.25
- We therefore say that the distribution is $X \sim B(10, 0.25)$

The calculation has three parts to it:



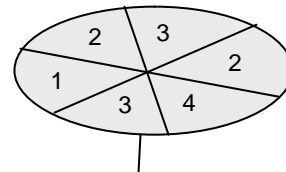
Thus there are always three parts to a binomial probability calculation *except* when you are at either end of the distribution. In which case: $P(\text{woken up all 10 nights}) = 0.25^{10}$; and the probability of not being woken up at all in ten nights is 0.75^{10} .

Many calculators will be able to calculate binomial probabilities for you. But you must enter the data in the correct order.

Actually, ${}^{10}C_{10} \times 0.25^{10} \times 0.75^0$

Getting the probability: You may simply be given the probability of success, or:

- You calculate the probability from previous experience (as in the example above)
- You calculate it from your knowledge of the situation (eg: success is getting a 2 on the spinner: $p = 1/3$).
- The probability is the result of a calculation from a previous part of the question.



350 of the 500 pupils in a school have the letter "s" in their name. If 6 pupils are chosen at random, what is the probability that 4 of them have an "s" in their name.

$P(\text{"s"}) = 350/500 = 0.7$. Therefore, $X \sim B(6, 0.7)$. $P(4 \text{ successes}) = {}^6C_4 \times 0.7^4 \times 0.3^2 = \underline{0.324}$

It is known that the probability of 2 successes in 4 trials is 0.3456. Find the value of p , the probability of success.

This will lead to a quartic equation – use your calculator to solve it.

YOU SOLVE

$p = 0.6$

More than one outcome: Since binomial probabilities are all mutually exclusive (I cannot be woken up both 3 nights *and* 4 nights in 10), the probability of one of several outcomes occurring can be found by addition. Thus, $P(\text{I am woken up 3 or 4 nights out of 10}) = {}^{10}C_3 \times 0.25^3 \times 0.75^7 + {}^{10}C_4 \times 0.25^4 \times 0.75^6 = 0.396$.